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HOW TO USE THE ORDINARY MATHEMATICAL MODELS IN THE MILITARY FIELD

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Abstract: *This article’s aim is to provide quick solutions in the case of military operations, more precisely it will be emphasized the fact that we can use mathematical algorithms of graph theory to determine the optimal length of roads in a mission. Specifically, the Elementary Algorithm (of Bellman) in the area of military operations in Iraq is used to determine the shortest route between two cities, Al Faw (a port situated in the south of Iraq) and Dahuk (in the north part of the country), through the capital, Baghdad. This is necessary for transporting troops and military equipment in specific areas. To achieve this, the map of Iraq is figured as a graph, the targeted cities are the vertices of the graph and are named as numbers (from 1 to 28), and distances between cities are the arcs of the graph (values, capacities – in this case are kilometers). The distances between cities and the country map are updated.*

Keywords: graph, optimal length, minimum length.

INTRODUCTION

This article refers to the modality in which the model of optimal length of paths in a graph can be applied or adapted to different situations in the military field, specifically in finding the optimal route between two locations using the algorithm for determining the optimal path length.

THEORETICAL CONSIDERATIONS

To solve such problems, elements of graph theory are used, in particular algorithms to determine the optimal length of roads.

In many practical problems we are faced with the situation to attach for each arc of the graph a number which could represent the travel time along the arc, the transport costs along the arc, the benefits, or the distance as in the problem bellow. Usually, in such a practical matter, the optimal path length (maximum and minimum) is required: the distance in time (as required), cost or capacity.

We will consider that the associated graph to this problem doesn’t have circuits, but it has an input node x_1 and an output node x_n.

ELEMENTARY ALGORITHM (OF BELLMAN)

This algorithm is based on the principle of optimality of Bellman, meaning that any optimal policy consists in optimal subpolicies.

According to this algorithm, to each node $x_i$is attached a number d_i representing the minimum length of paths from x_1 to $x_i$measured as distance, cost, fuel consumption, distance in time, number of people in work / operation (download, upload), quantity of food required for crew between ports.

We consider $d_1 = 0$. Now, suppose that we want to find d_l, where x_l is the successor of nodes x_i, x_j and x_k till where the numbers d_i, d_j and d_k were already calculated. Then, the minimum length d_l from x_1 to $x_l$is determined by the formula

$$d_l = \min(d_i + c_{il},\ d_j + c_{jl},\ d_k + c_{kl}) \qquad (1)$$

where c_{il}, c_{jl} and c_{kl} are appropriate capabilities of the arcs (x_i, x_l) (x_j, x_l) and (x_k, x_l).In the formula of d_l we will underline in the parentheses the value for which the minimum is reached.

After determining all the numbers $d_1, d_2, . . . , d_n$, the value of $d_l$is the minimum length of the path from x_1 to x_n and starting from x_n to $x_1$and reading the underlined nodes, we get the minimum path length.

For a maximum length road we could proceed absolutely in the the same way, but we replace the minimum with the maximum [1], [2].

APPLYING THE MODEL OF OPTIMAL LENGTH OF PATHS IN THE MILITARY FIELD

We will further illustrate how the model of optimal length of paths in a graph can be applied or adapted to different situations in the military field.

Example: Determining the optimal route between two localities.

We imagine that the commander of a military structure deployed in Iraq as part of a NATO mission (for establishing order and maintaining peace in the operational area), has to determine the optimal route (minimum) movement of a mobile subunit between the port Al Faw, located in South, bordering the Persian Gulf and the town of Dahuk, situated in the North. Subunit consists of military forces and military means - trucks

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transporting certain materials of military equipment from Al Faw to Baghdad, and in Baghdad they will be taken and will be escorted by a troop of Romanian infantrymen and a troop of US Marines to the town of Dahuk.
The commander of the military structure has been provided a map of the area where the operation is to be performed, shown in Figure 1 below. Using the data and information held by the unit management and the topographical measurements existant, the unit specialists were able to determine road distances between the target towns listed on the map. Also, the routes were chosen taking into account the security forces and means, knowing that terrorist attacks are commonplace.
Based on this information, it is desired to determine how the movement should be organized between localities Al Faw and Dahuk so that the subunit reach in a short time the destination and pass through the town of Baghdad.

Figure 1. Subunit area of military operations[5]
Appropriate distances between localities in the operations area are given in the table below [4]:

Departure		Arrival		Distance (km)
Node no	**City's name**	**Node no**	**City's name**	
(1)	Al Faw	**(2)**	Um Qasr	29.50
		(3)	Al Basrah	72.62
		(14)	Ali al Gharbi	363.37
(2)	Um Qasr	**(3)**	Al Basrah	63.53
		(4)	Zubayr	54.90
		(5)	An Nashirayah	277.28
(3)	Al Basrah	**(5)**	An Nashirayah	231.96
		(7)	Al Amarah	181.35
		(14)	Ali al Gharbi	237.24
(4)	Zubayr	**(5)**	An Nashirayah	246.32
		(6)	Suq ash Shuyukh	269.40
		(8)	As Samawah	297.48
(5)	An Nashirayah	**(7)**	Al Amarah	147.84
		(8)	As Samawah	110.17
		(9)	Ash Shatrah	45.83
(6)	Suq ash Shuyukh	**(5)**	An Nashirayah	31.24
		(8)	As Samawah	157.16
		(11)	An Najaf	312.70
		(15)	Nukhayb	554.38
(7)	Al Amarah	**(9)**	Ash Shatrah	172.67
		(10)	Al Hayy	150.70
		(13)	Al Kut	196.91
		(25)	As Sulaymaniya	630.01
(8)	As Samawah	**(11)**	An Najaf	157.24
		(18)	Baqubah	334.73
(9)	Ash Shatrah	**(8)**	As Samawah	148.69
		(10)	Al Hayy	94.35
		(11)	An Najaf	255.90
(10)	Al Hayy	**(12)**	All Hillah	155.71
		(13)	Al Kut	48.22
(11)	An Najaf	**(12)**	All Hillah	59.32
		(15)	Nukhayb	266.28
		(21)	Ramadi	219.56
(12)	All Hillah	**(16)**	Karbala	48.27
		(17)	Baghdad	117.54
		(25)	As Sulaymaniya	476.71
(13)	Al Kut	**(12)**	All Hillah	169.00
		(18)	Baqubah	217.52
		(25)	As Sulaymaniya	445.91
(14)	Ali al Gharbi	**(13)**	Al Kut	109.56
		(20)	Khamaqin	346.54
(15)	Nukhayb	**(21)**	Ramadi	290.70
		(22)	Al Hadithah	365.89
(16)	Karbala	**(17)**	Baghdad	103.85
		(21)	Ramadi	141.02
(17)	Baghdad	**(18)**	Baqubah	64.26
		(19)	Samarra	134.64
		(23)	Anah	320.37
(18)	Baqubah	**(21)**	Ramadi	182.29
		(24)	Kirkuk	212.86

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Table 1. Road distances between towns in the area of operations

Departure		Arrival		Distance (km)
Node no	**City's name**	**Node no**	**City's name**	
(19)	Samarra	**(21)**	Ramadi	123.37
		(23)	Anah	305.95
		(24)	Kirkuk	170.09
		(27)	Al Mawsil (Mosul)	292.59
(20)	Khamaqin	**(24)**	Kirkuk	203.13
		(25)	As Sulaymaniya	228.54
(21)	Ramadi	**(22)**	Al Hadithah	148.95
		(23)	Anah	201.27
(22)	Al Hadithah	**(23)**	Anah	72.96
(23)	Anah	**(27)**	Al Mawsil (Mosul)	396.04
		(28)	Dahuk	532.05
(24)	Kirkuk	**(23)**	Anah	314.62
		(25)	As Sulaymaniya	104.25
		(26)	Erbil	98.40
(25)	As Sulaymaniya	**(26)**	Erbil	183.39
		(28)	Dahuk	374.85
(26)	Erbil	**(27)**	Al Mawsil (Mosul)	85.02
		(28)	Dahuk	211.58
(27)	Al Mawsil (Mosul)	**(28)**	Dahuk	139.45
(28)	Dahuk			

Table 2. Road distances between towns in the area of operations

Based on the map in Figure 1 and Tables 1 and 2, we may draw the graph corresponding to the area of operations, represented like in Figure 2:

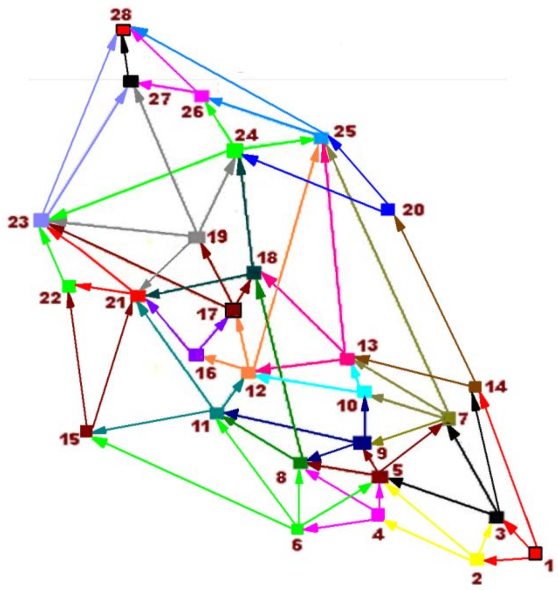

Figure 2. Corresponding Graph for the area of operations

For the first part of the route, between the towns Al Faw and Baghdad, the route of optimal length is shown by the following calculations:

$\underline{d_1}= 0$,
$\underline{d_2}= \min\{\underline{d_1+ 29.50}\} = \min\{29.50\} = \underline{29.50}$;
$\underline{d_3}= \min\{d_1+ 72.62, \underline{d_2+ 63.53}\} = \min\{\underline{72.62}, 93.03\} = \underline{72.62}$;
$d_4= \min\{d_2+ 54.90\} = \min\{84.40\} = 84.40$;
$d_5= \min\{d_3+ 231.96, d_4+246.32, d_2+277.28\} = \min\{304.58, 330.72, 306.78\} = 304.58$;
$d_6= \min\{d_4+ 269.40\} = \min\{353.80\} = 353.80$;
$\underline{d_7} = \min\{\underline{d_3+ 181.35}, d_5+147.84\} = \min\{\underline{253.97}, 457.42\} = \underline{253.97}$;
$d_9= \min\{d_5+ 45.83, d_7+ 172.67\} = \min\{350.41, 433.64\} = 350.41$;
$d_8= \min\{d_4+ 297.48, d_5+ 110.17, d_6+ 157.16, d_9+148.69\} = \min\{381.88, 414.75, 510.96, 499.10\} = 381.88$;
$\underline{d_{10}}= \min\{\underline{d_7+ 150.70}, d_9+ 94.35\} = \min\{\underline{404.67}, 444.76\} = \underline{404.67}$;
$d_{11}= \min\{d_6+ 312.70, d_8+157.24, d_9+ 255.90\} = \min\{666.50, 539.12, 606.31\} = 539.12$.
$d_{14}= \min\{d_1+ 363.37, d_3+ 237.24\} = \min\{363.37, 309.86\} = 309.86$;
$d_{13}= \min\{d_7+ 196.91, d_{10}+ 48.22, d_{14}+ 109.56\} = \min\{450.88, 452.89, 419.42\} = 419.42$;
$\underline{d_{12}}= \min\{\underline{d_{10}+ 155.71}, d_{11}+ 59.32, d_{13}+ 169.00\} = \min\{\underline{560.38}, 598.44, 588.42\} = \underline{560.38}$;
$d_{15}= \min\{d_6+ 554.38, d_{11}+ 266.28\} = \min\{908.18, 805.40\} = 805.40$;
$d_{16}= \min\{d_{12}+ 48.27\} = \min\{608.65\} = 608.65$;
$d_{17}= \min\{\underline{d_{12}+ 117.54}, d_{16}+ 103.85\} = \min\{\underline{677.92}, 712.50\} = \underline{677.92}$.

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Therefore, the minimum length for the distance between localities Al Faw (1) and Baghdad (17) is 677.92 km, and the route which has this length is:

$dmin(1\text{-}17) = \{x_1, x_2, x_3, x_7, x_{10}, x_{12}, x_{17}\}$.

Therefore, the path of minimum length for the first part of the route is:

Al Faw – Um Qasrr - Basrah - Al Amarah – Al Hayy – Al Hillah – Baghdad with a length of 677.92 km.

Further, for the second part of the route, we will select the starting node(17), corresponding to the city of Baghdad and the final destination node (28), corresponding to the city of Dahuk. The optimal solution is calculated as follows:

$\underline{d_{17}} = 0$,

$\underline{d_{18}} = \min\{\underline{d_{17} + 64.26}\} = \min\{\underline{64.26}\} = \underline{64.26}$;

$d_{19} = \min\{d_{17} + 134.64\} = \min\{134.64\} = 134.64$;

$d_{20} = \min\{d_{14} + 346.54\} = \min\{656.40\} = 656.40$;

$d_{21} = \min\{d_{11} + 219.56, d_{15} + 290.70, d_{16} + 141.02, d_{18} + 182.29\} = \min\{758.68, 1096.10, 749.67, 246.55\} = 246.55$;

$d_{22} = \min\{d_{15} + 365.89, d_{21} + 148.95\} = \min\{1171.29, 395.50\} = 395.50$;

$\underline{d_{24}} = \min\{\underline{d_{18} + 212.86}, d_{19} + 170.09, d_{20} + 203.13\} = \min\{\underline{278.12}, 304.73, 859.53\} = \underline{278.12}$;

$d_{23} = \min\{d_{17} + 320.37, d_{19} + 305.95, d_{21} + 201.27, d_{22} + 72.96, d_{24} + 314.62\} = \min\{320.37, 440.59, 447.82, 468.46, 592.74\} = 320.37$;

$d_{25} = \min\{d_7 + 630.01, d_{12} + 476.71, d_{13} + 445.91, d_{20} + 228.54, d_{24} + 104.25\} = \min\{883.98, 1037.09, 865.33, 884.94, 382.37\} = 382.37$;

$\underline{d_{26}} = \min\{\underline{d_{24} + 98.40}, d_{25} + 183.39\} = \min\{\underline{376.52}, 565.76\} = \underline{376.52}$;

$d_{27} = \min\{d_{19} + 292.59, d_{23} + 396.04, d_{26} + 85.02\} = \min\{970.51, 716.41, 461.54\} = 461.54$;

$d_{28} = \min\{d_{23} + 532.05, d_{25} + 374.85, \underline{d_{26} + 211.58}, d_{27} + 139.45\} = \min\{852.42, 757.22, \underline{588.10}, 600.99\} = \underline{588.10}$.

Therefore, the minimum length between the towns Baghdad (17) and Dahuk (28) is 588.10 km, and the route which has this length is:

$dmin(17\text{-}28) = \{x_{17}, x_{18}, x_{24}, x_{26}, x_{28}\}$.

In such circumstances the minimum path length for the second part of the route is:

Baghdad – Baqubah - Kirkuk – Erbil – Dahuk with a length of 588.10 km.

So, the minimal length of the route between the localities of Al Faw and Dahuk which goes through Baghdad is:

$dmin(1\text{-}28) = \{x_1, x_2, x_3, x_7, x_{10}, x_{12}, x_{17}, x_{18}, x_{24}, x_{26}, x_{28}\}$, namely:

$dmin(1\text{-}17) + dmin(17\text{-}28) = 677.92 + 588.10 = 1266.02$

It means:Al Faw – Um Qasrr - Basrah - Al Amarah – Al Hayy – Al Hillah – Baghdad – Baqubah - Kirkuk – Erbil – Dahuk with a length of 1266.02 km.

In Figure 3 the minimum path arches are doubled by dashes.

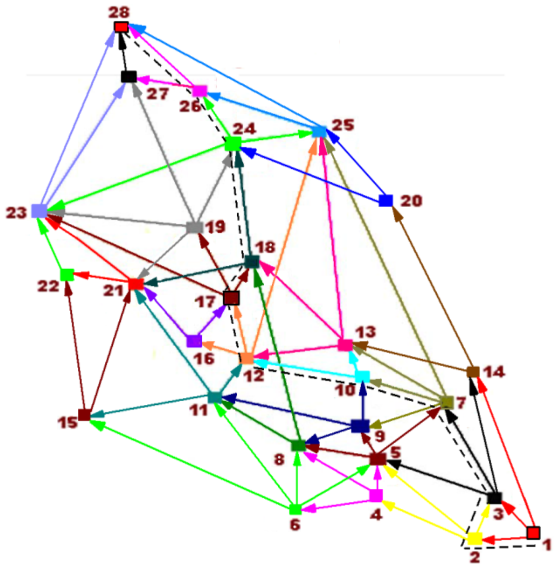

Figure 3. The representation of the minimal route corresponding to the operational area

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